Single-Player Hanabi

not as boring as you'd think!

Cards

ϑ = # values

C = # colors

color

a single card $\rightsquigarrow$ (p, q)

value

The Single-Player version

Given : a stream of N cards .

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(identical cards might repeat up to r times)

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Moves : Discard, Play, Store

can play (p, q) iff
the last card of color q played had value p-1
or p = 1 & no cards of color q have been played.

The Single-Player version

Given : a stream of N cards.

(identical cards might repeat up to r times)

Moves : Discard, Play, Store

Win :

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for all colors c

The Single-Player version

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Moves : Discard, Play, Store

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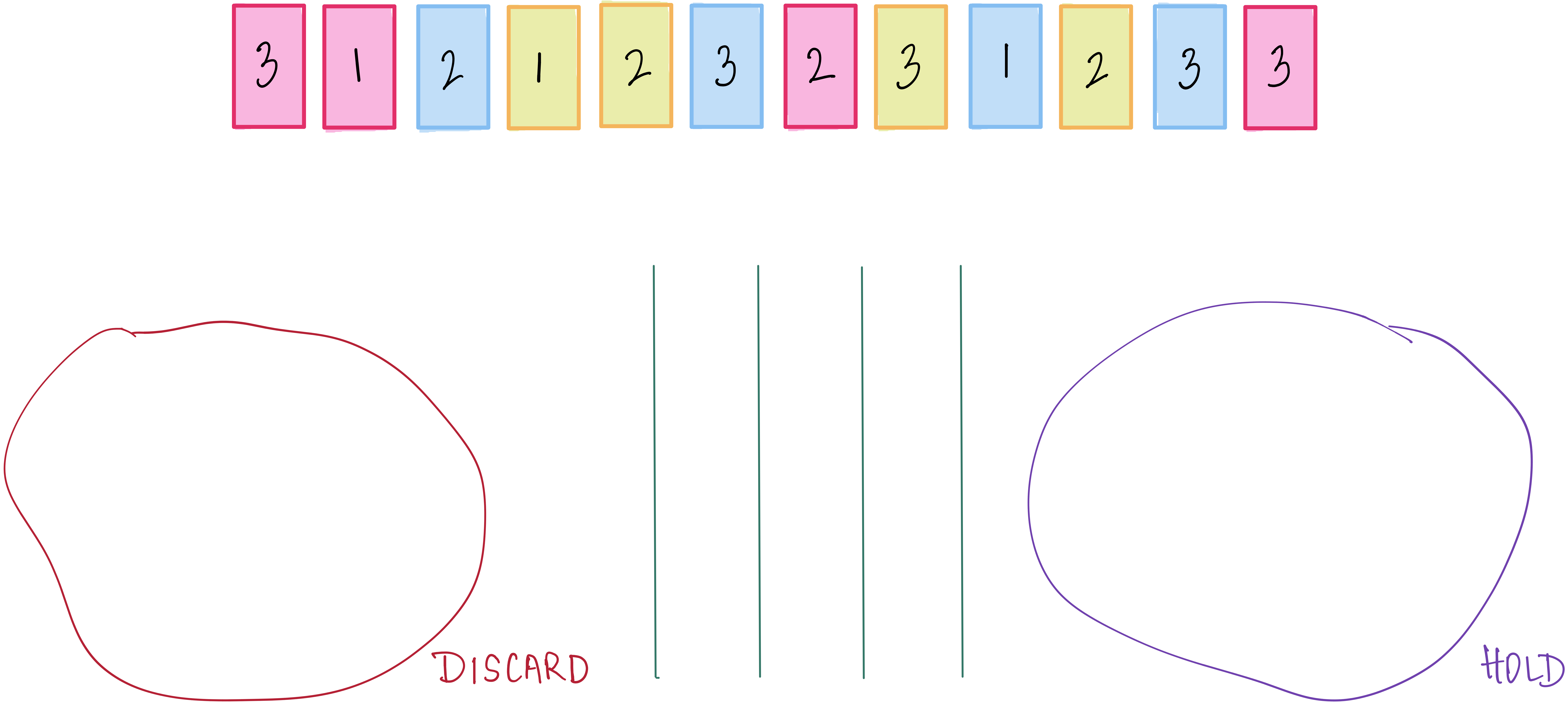
for all colors c

while storing $\leq h$ cards @ all times

Example Run.

$N=12$; $v=3$, $c=3$

$r=2$, $h=2$



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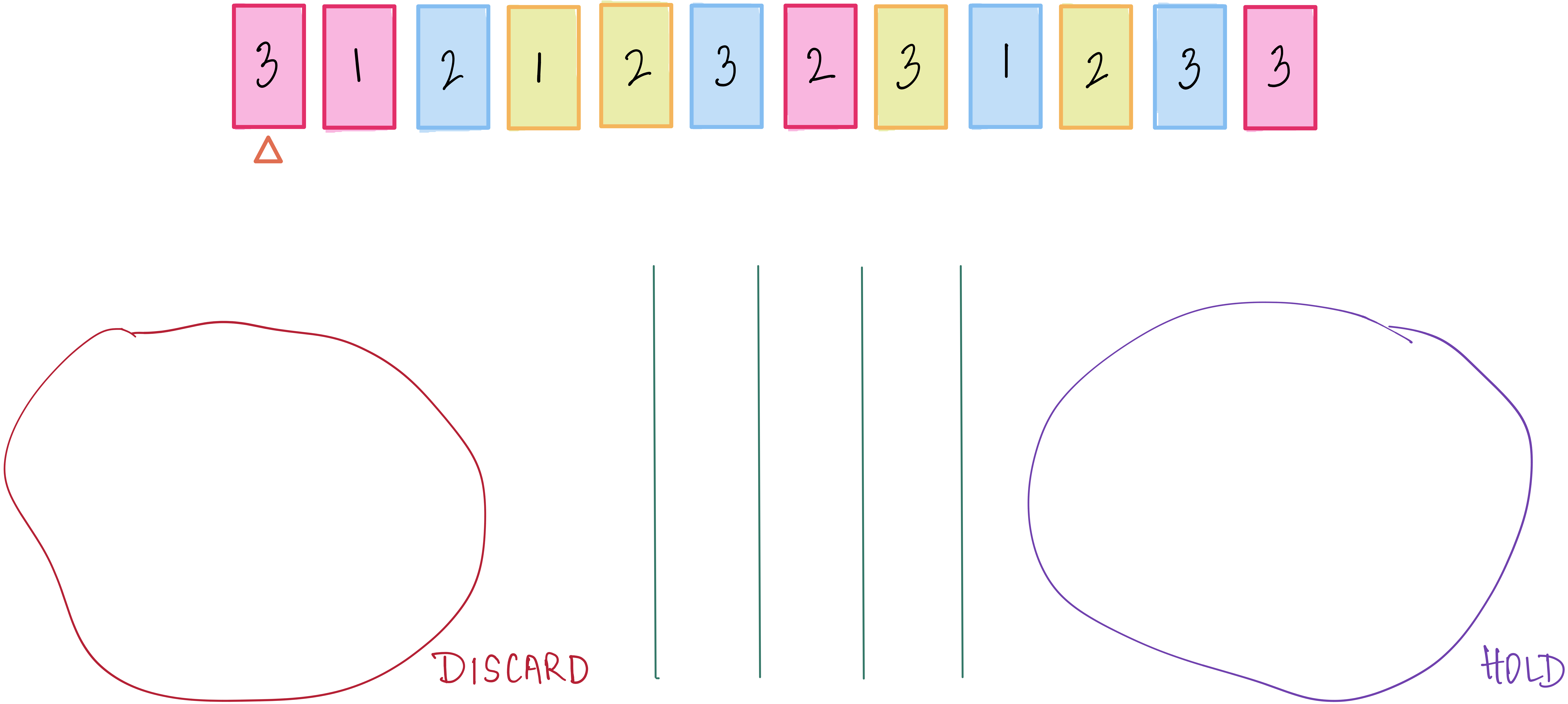

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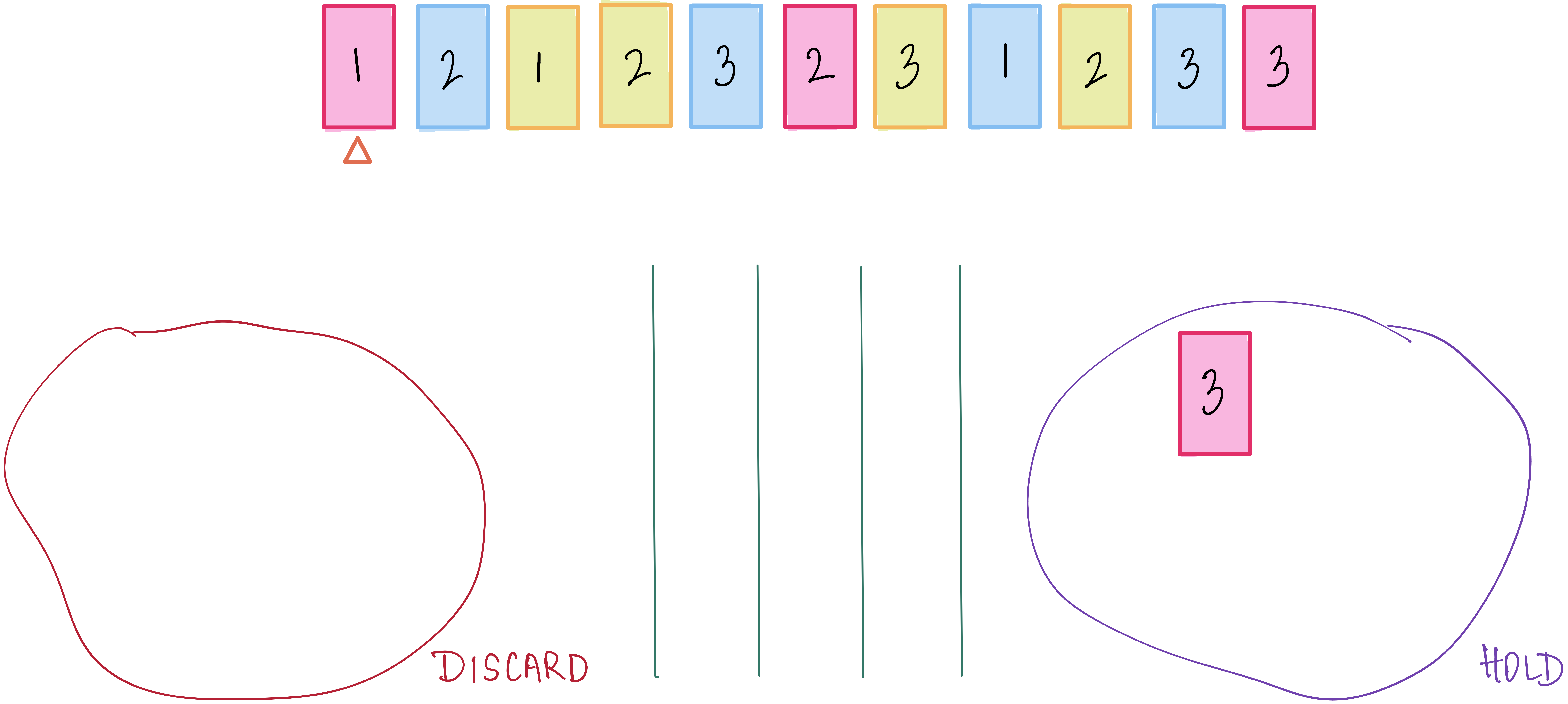

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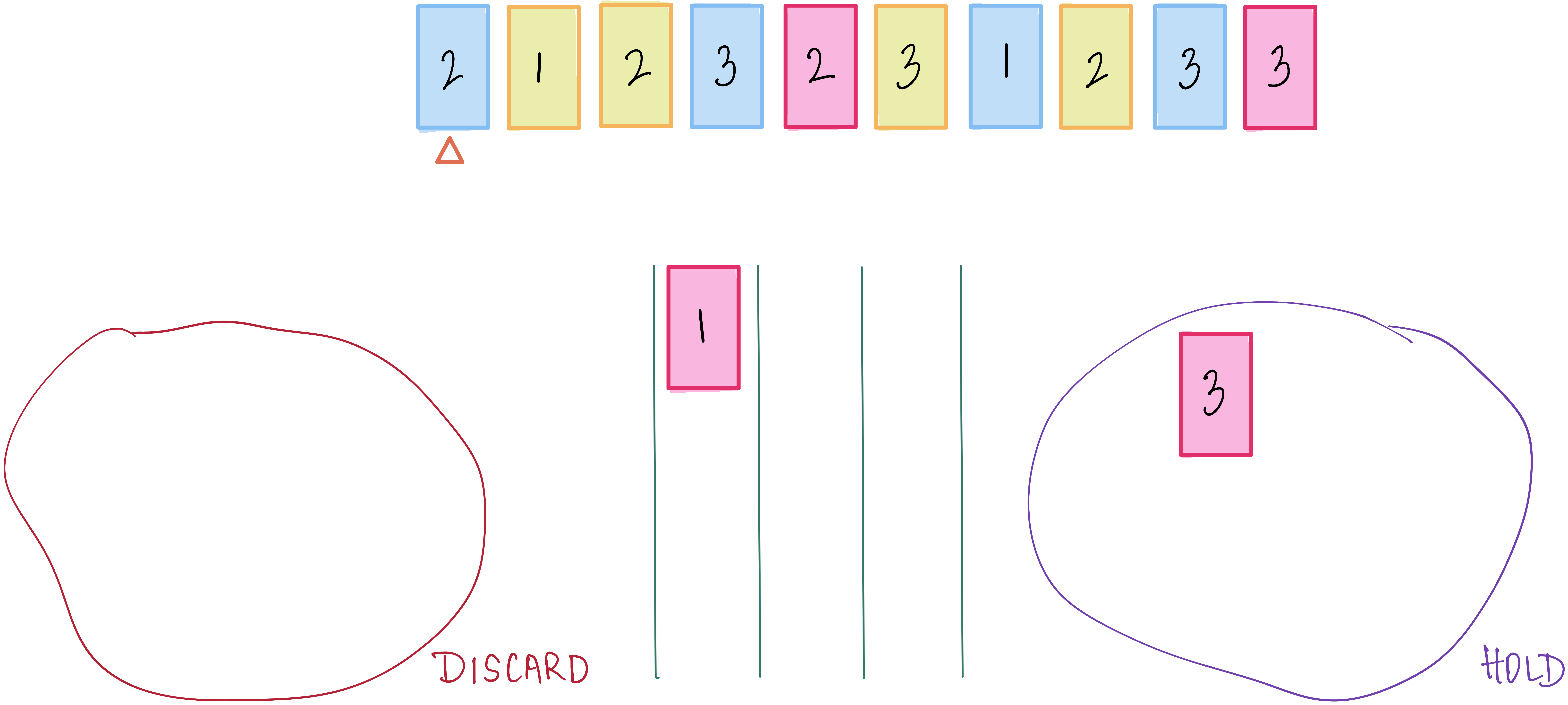

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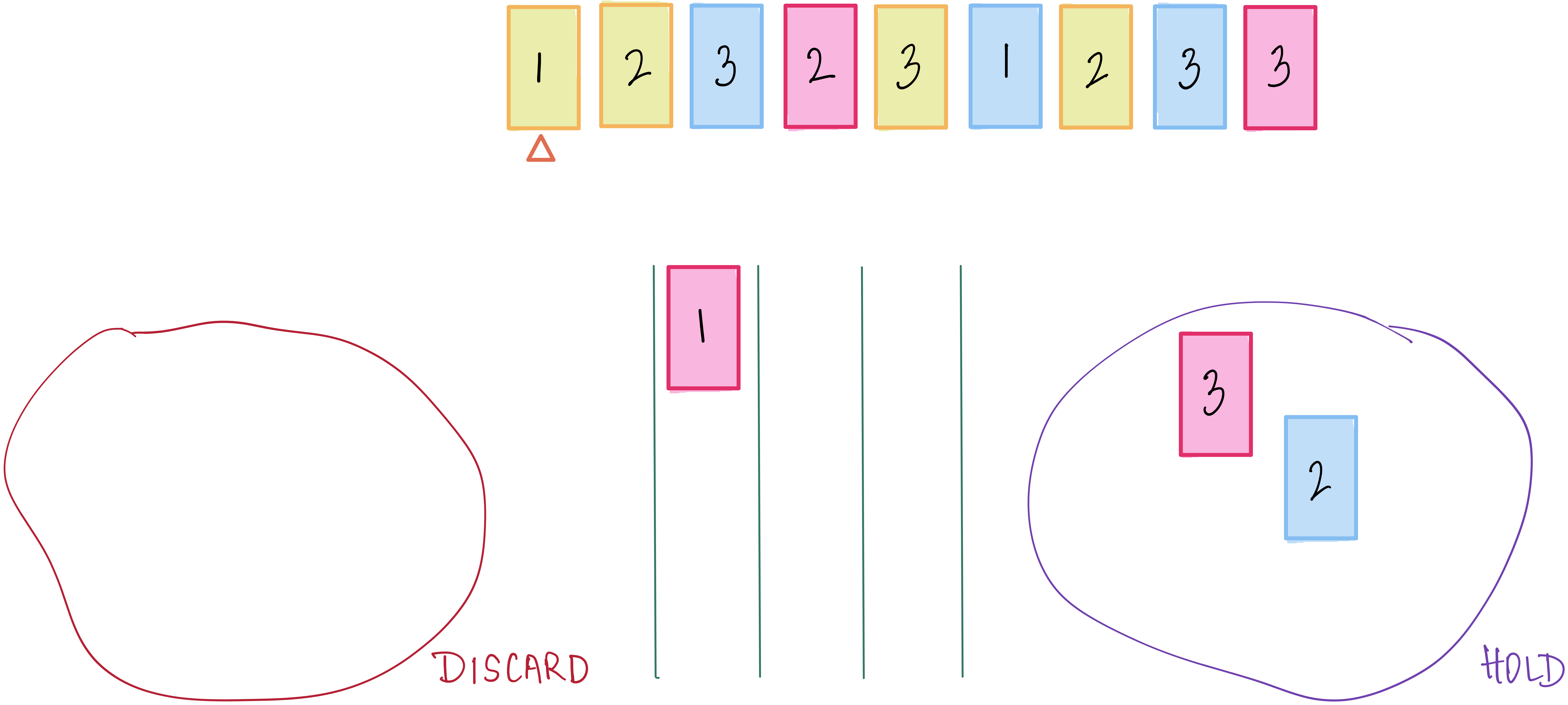

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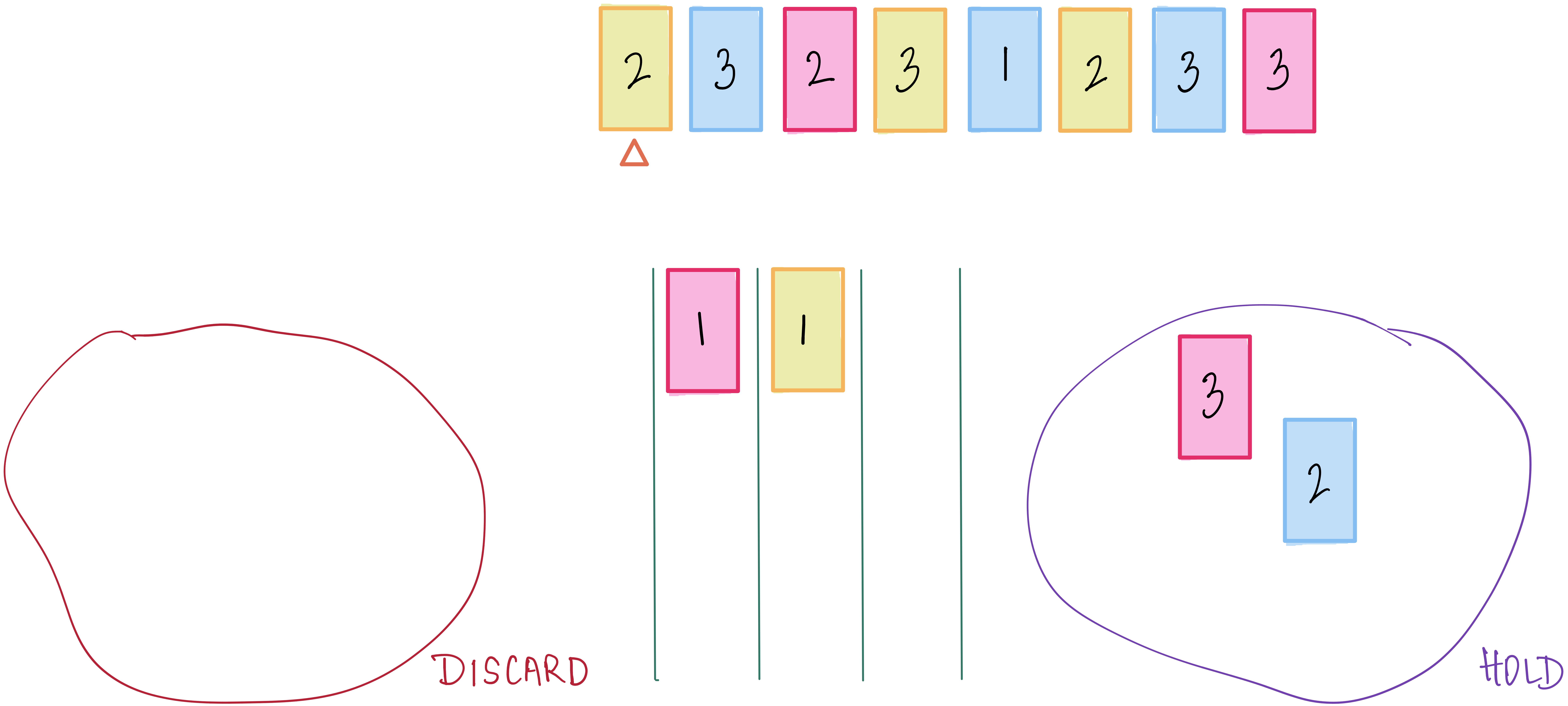

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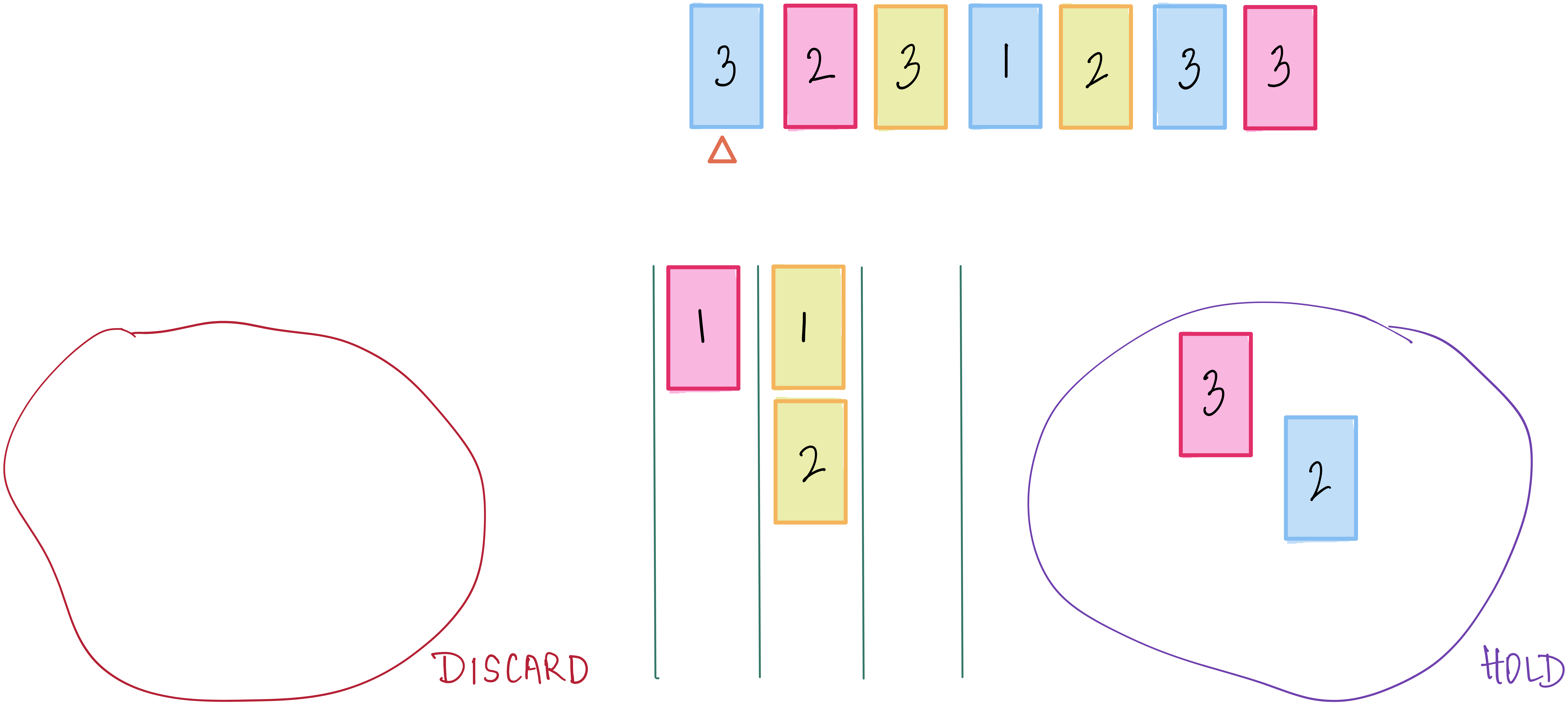

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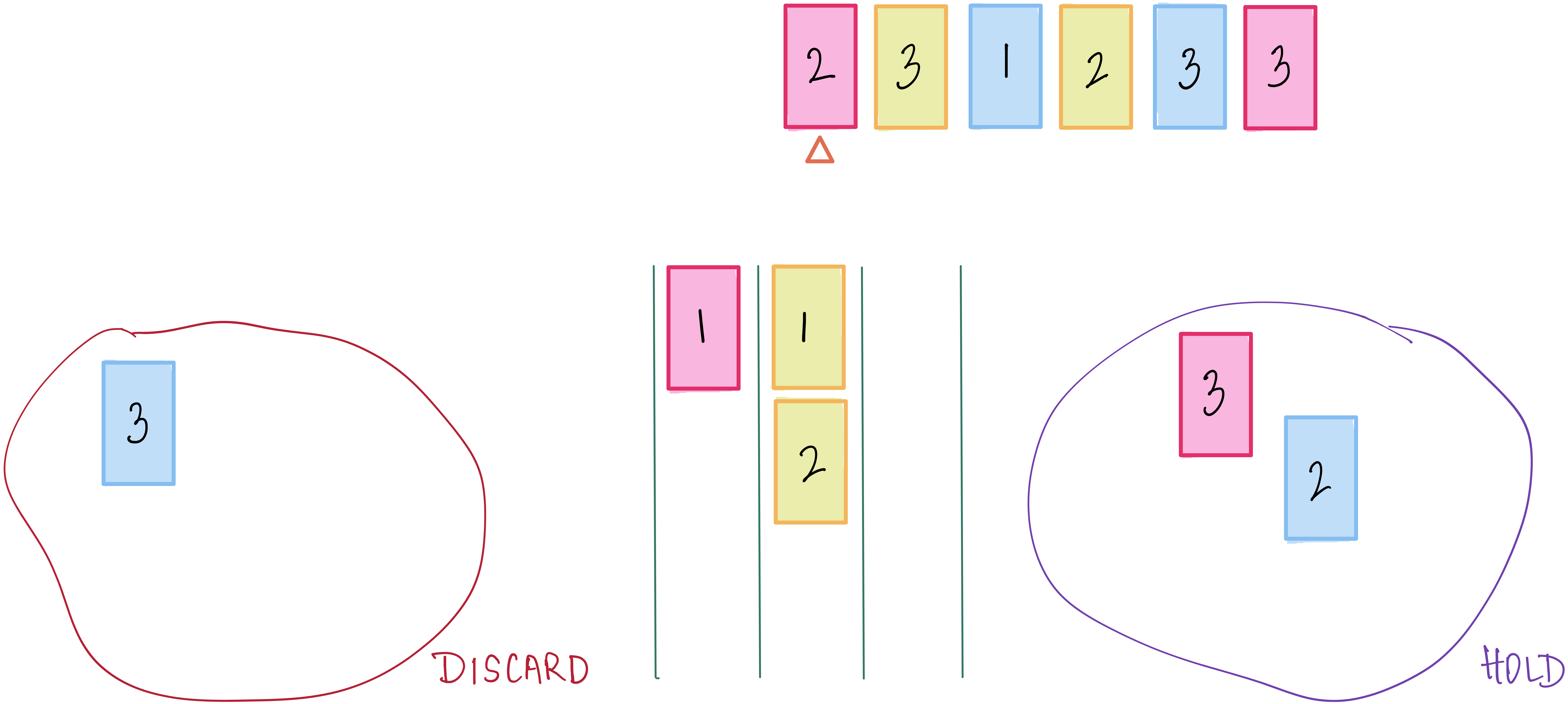

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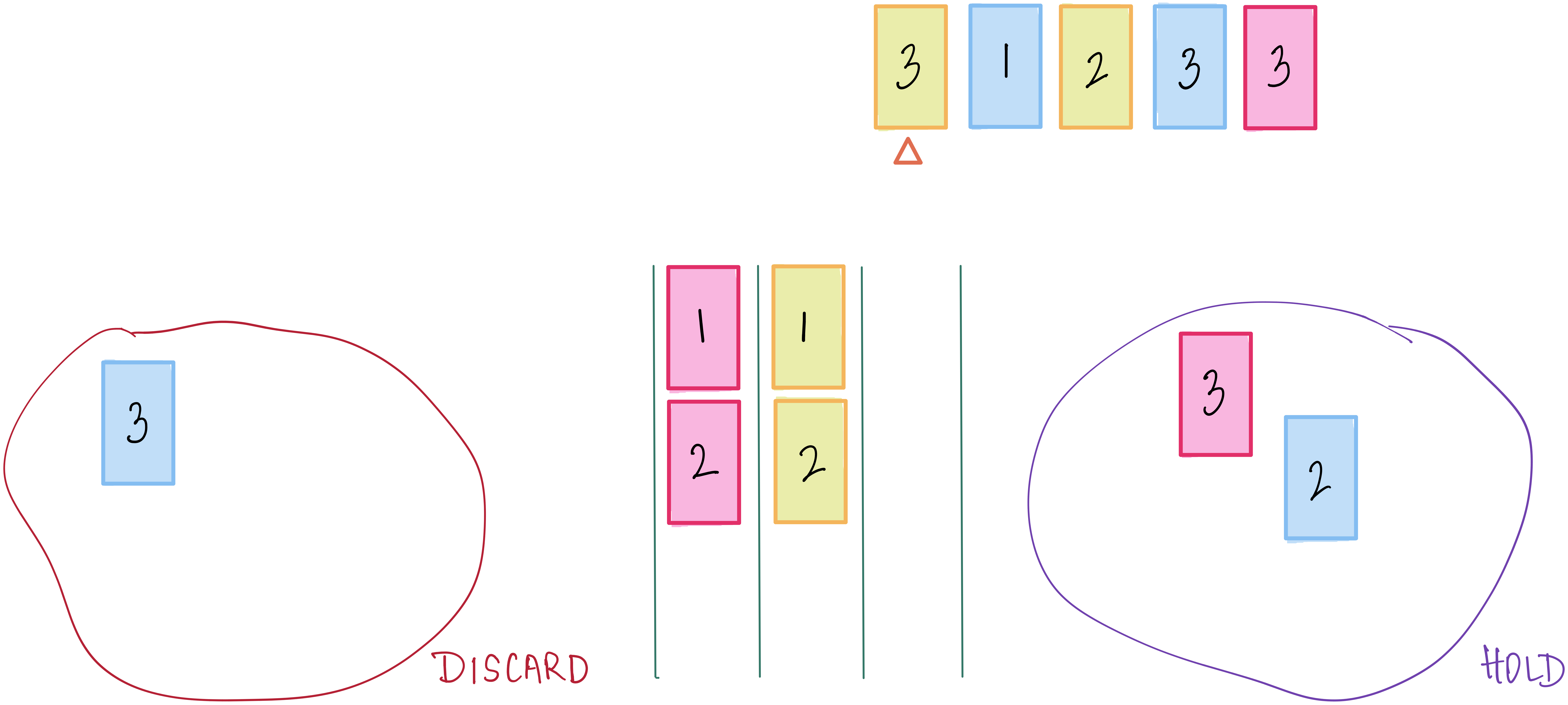

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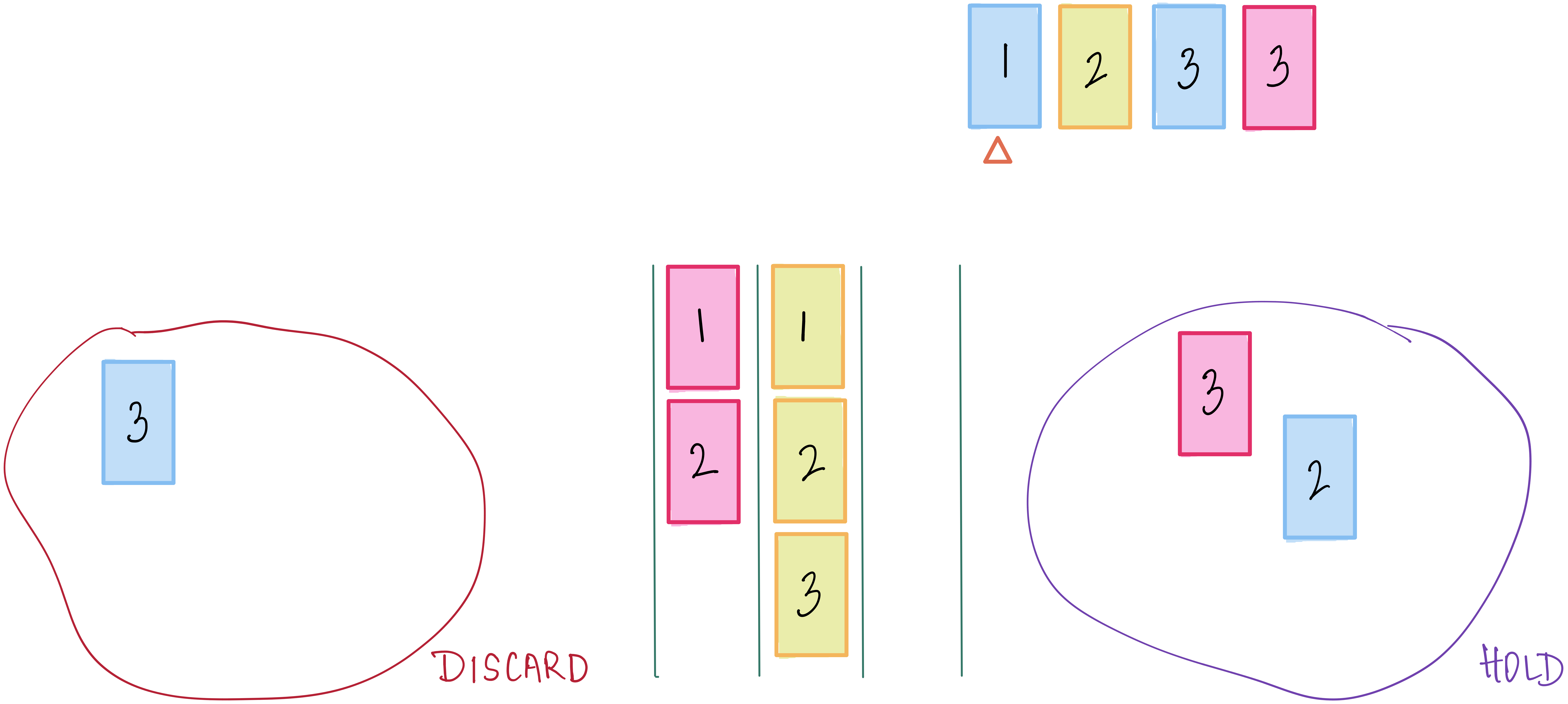

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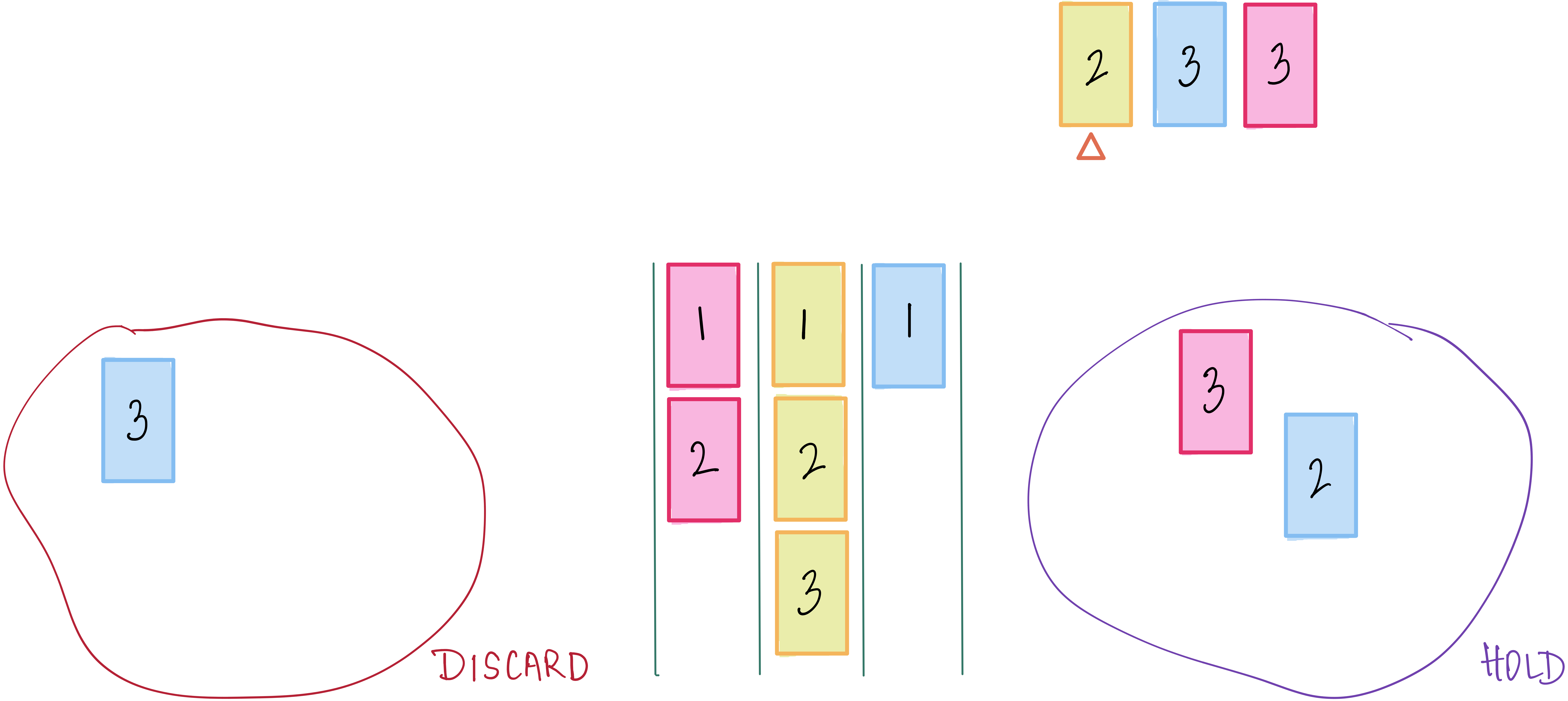

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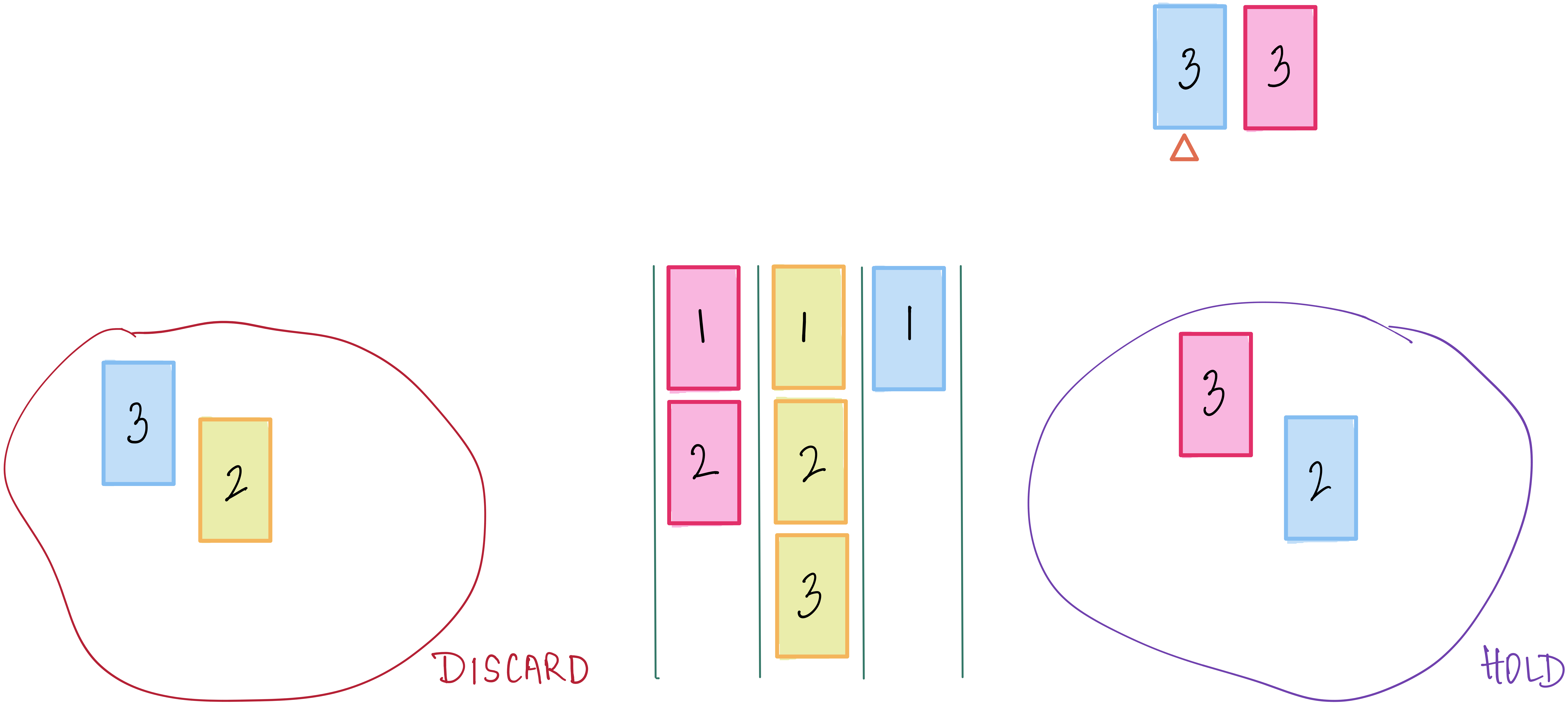

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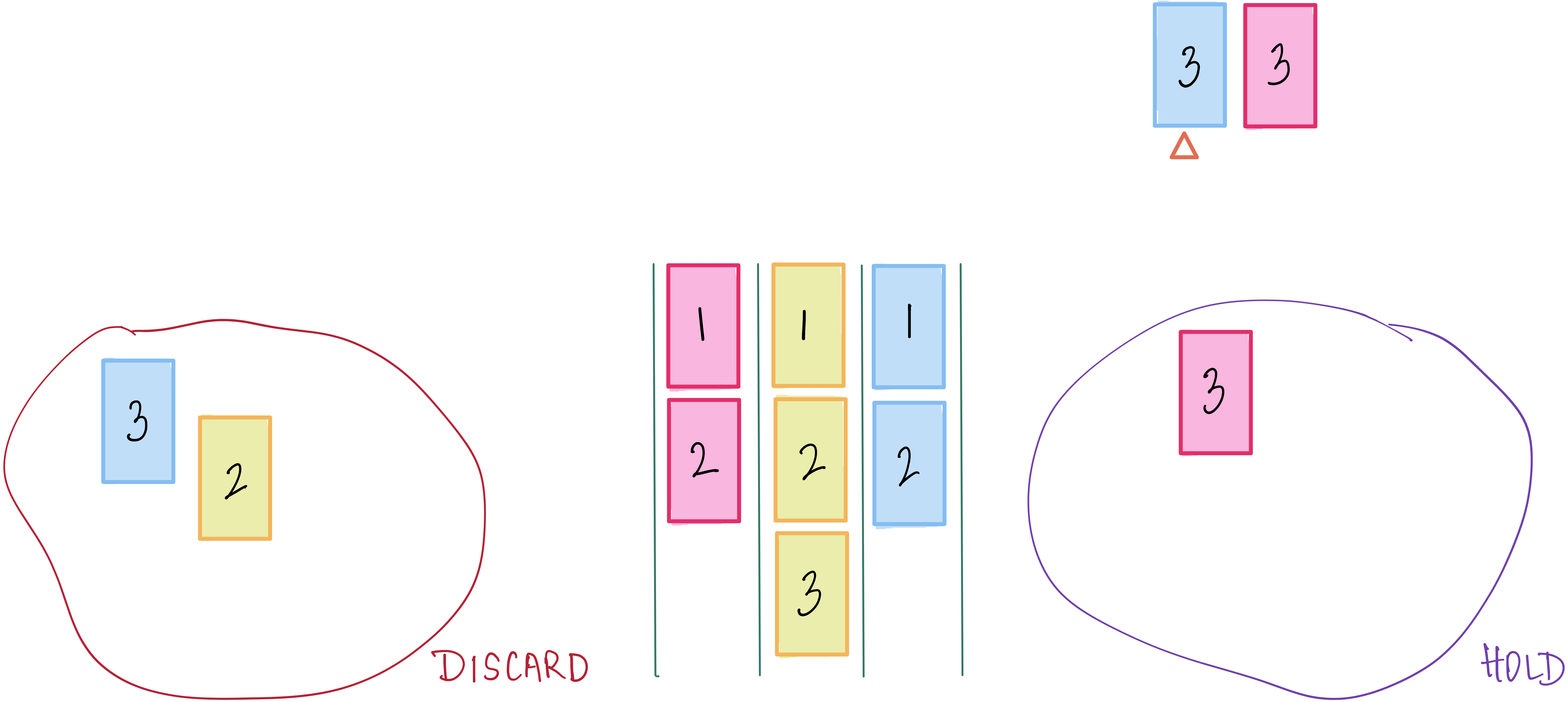

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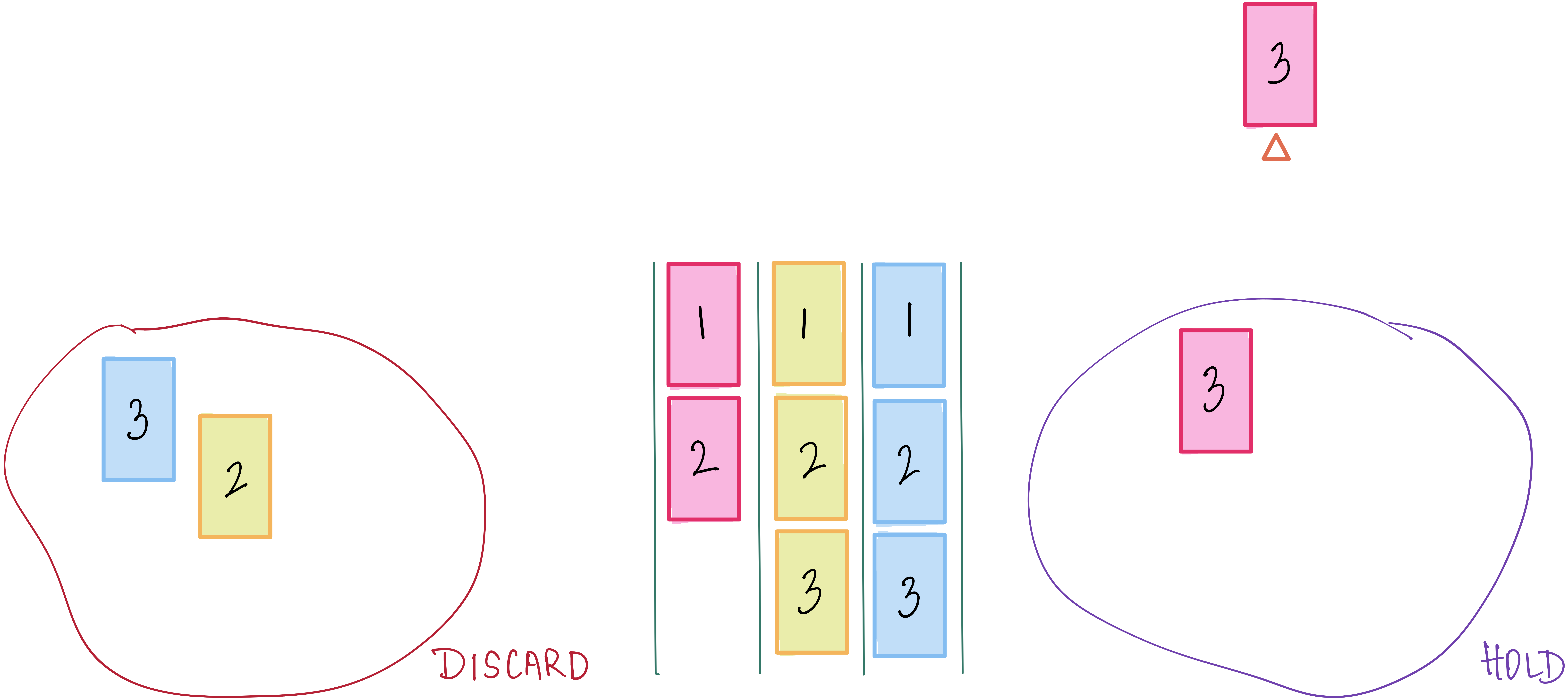

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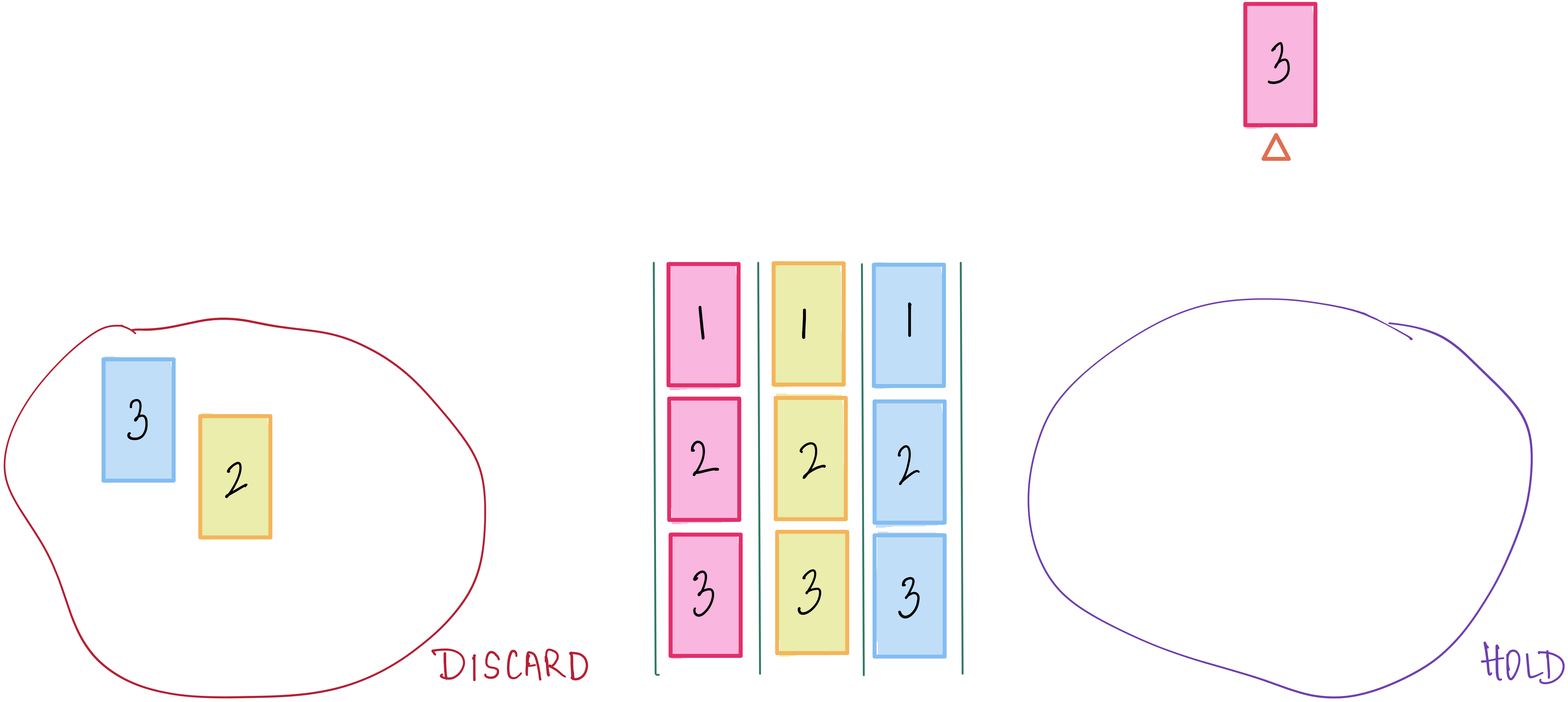

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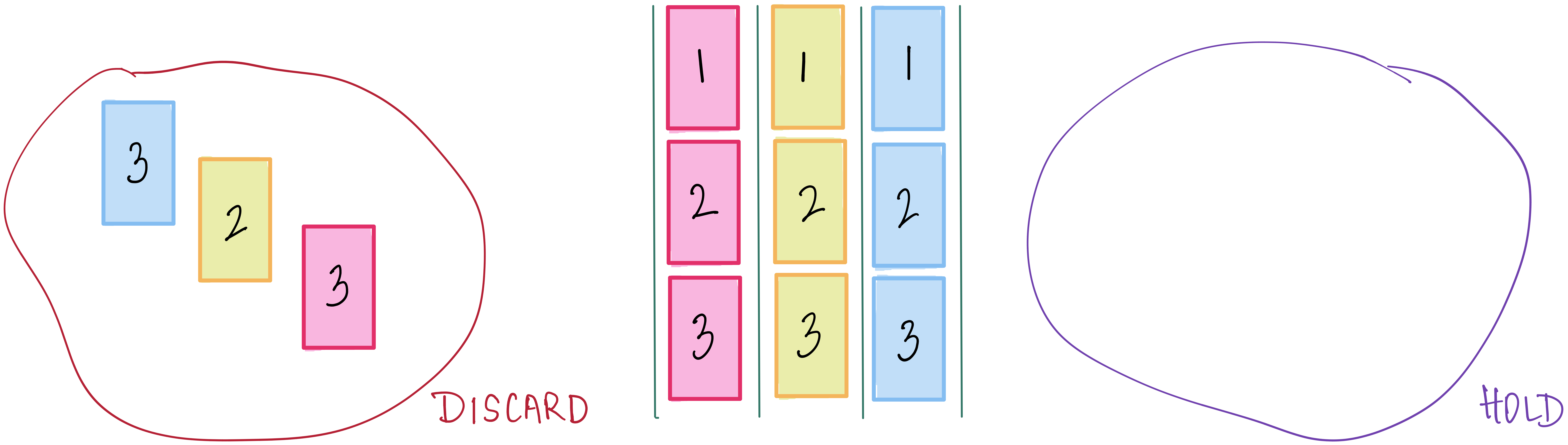

Special Cases

$\underbrace{r = 1}$

no repeats.

What is the $\underbrace{\text{smallest holding threshold}}_{h}$ with which we can win?

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Q1. How much holding power do you need for this to work?

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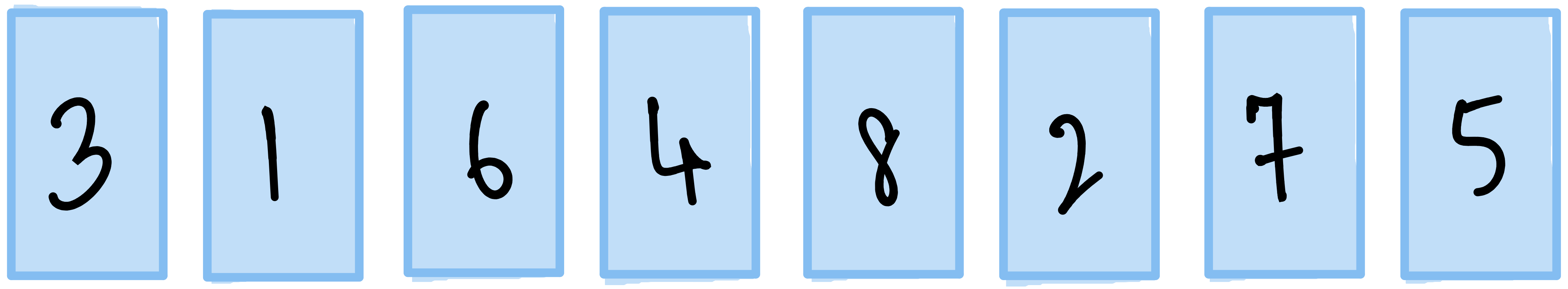
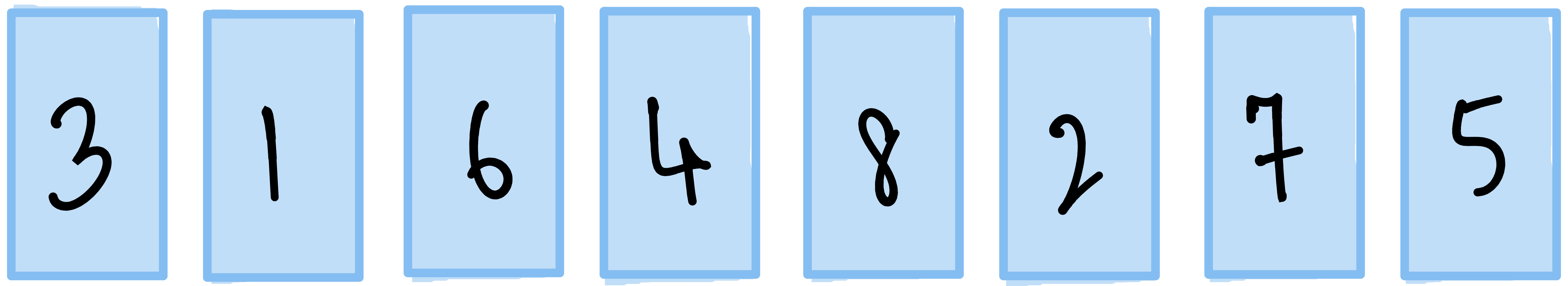
What is the smallest $\underbrace{\text{holding threshold}}_{h}$ with which we can win?

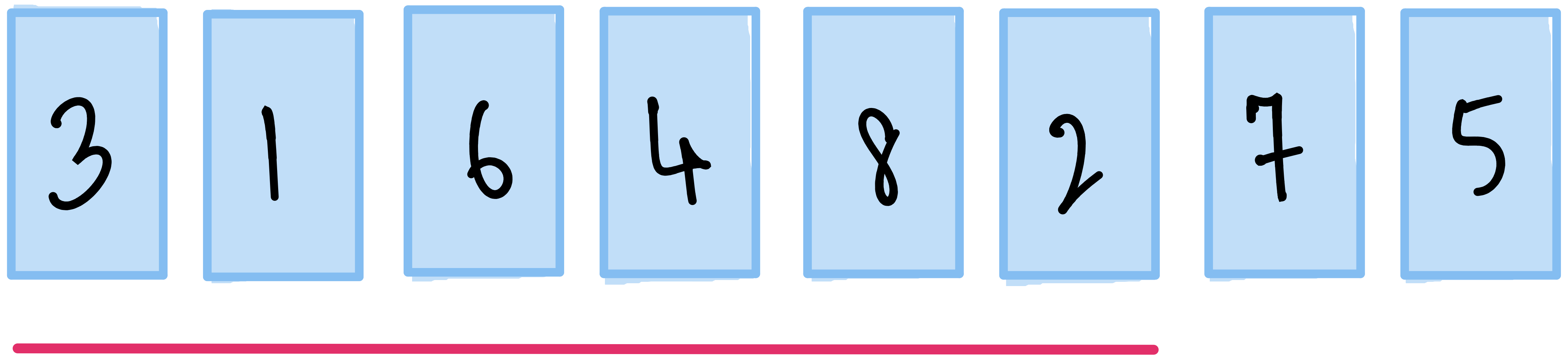
Natural strategy: play a card as early as you can.

Q2. Can you do better?


3
1
6
4
8
2
7
5



Draw an interval from every card to its rightmost card w/ same color but smaller value



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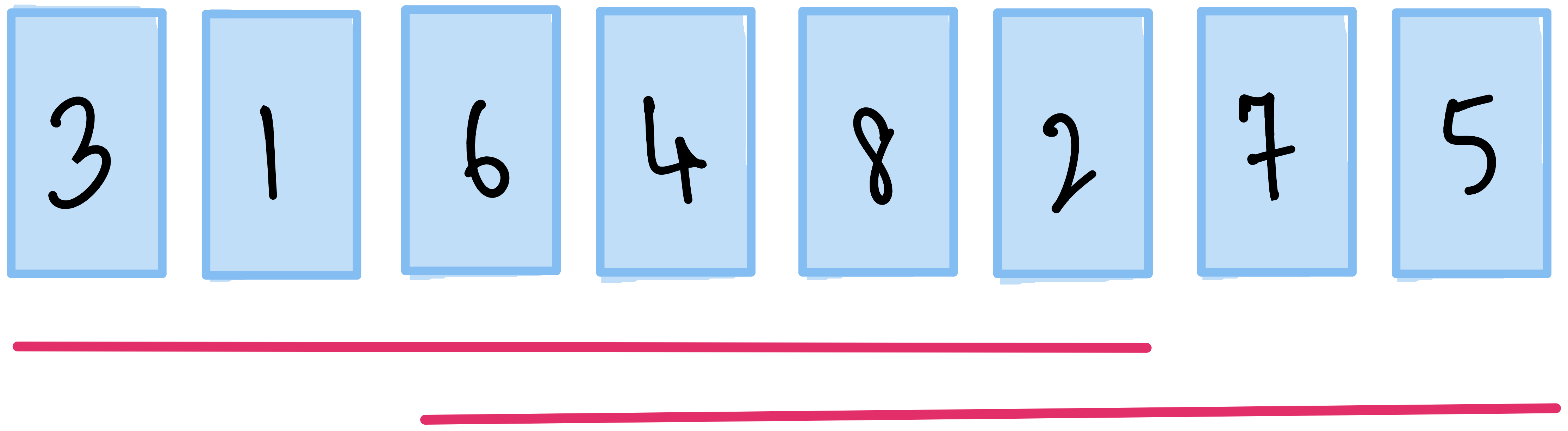

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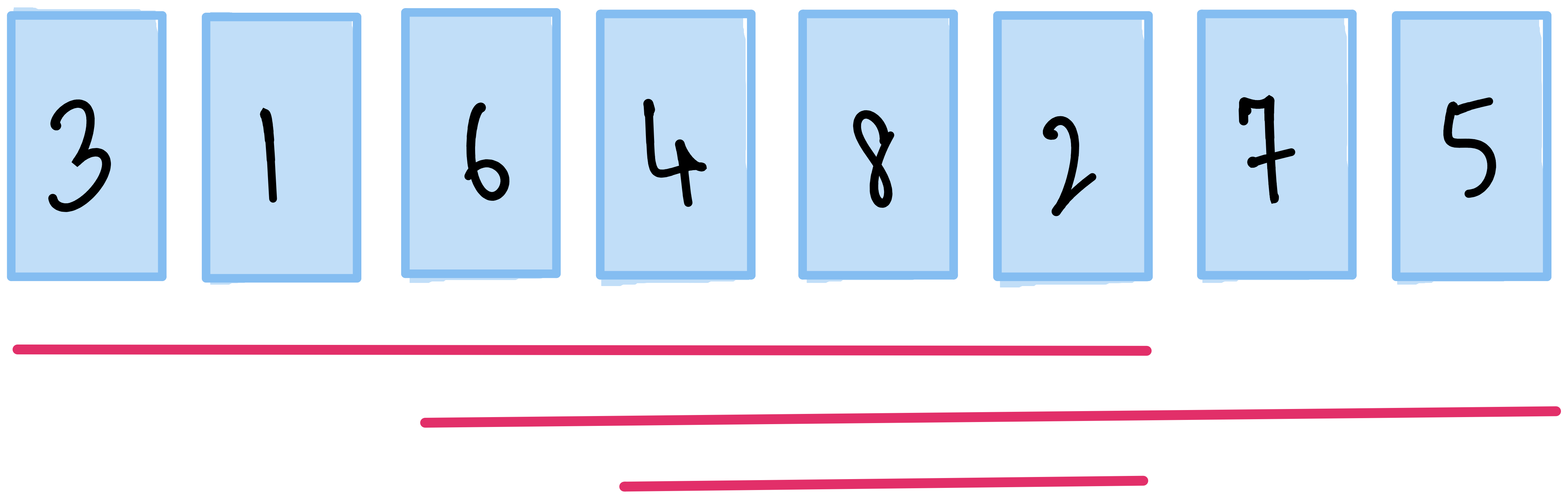

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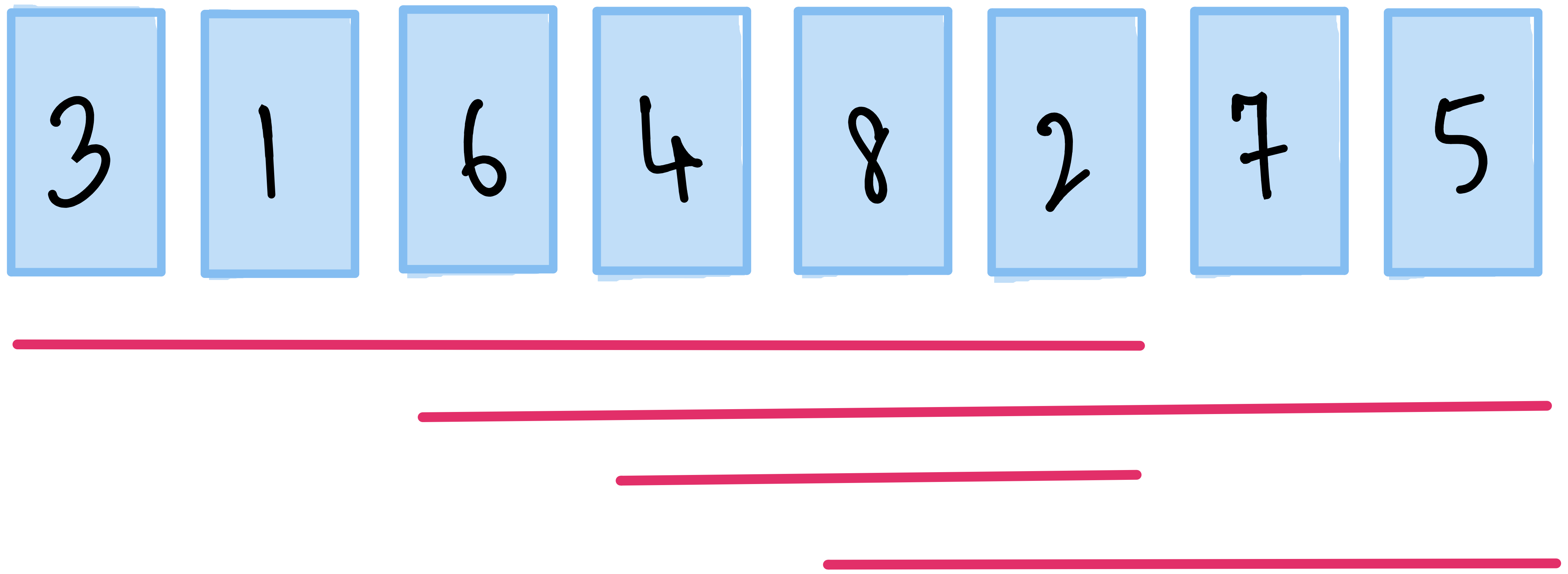

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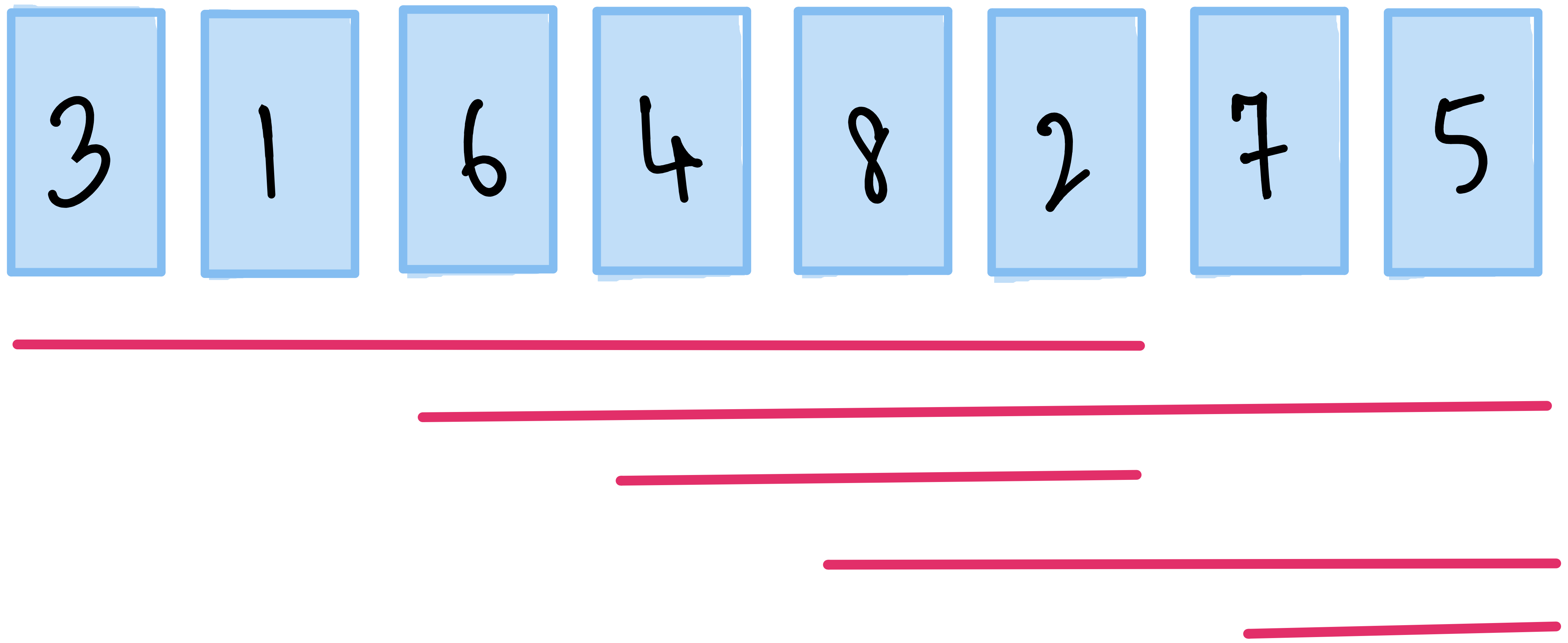

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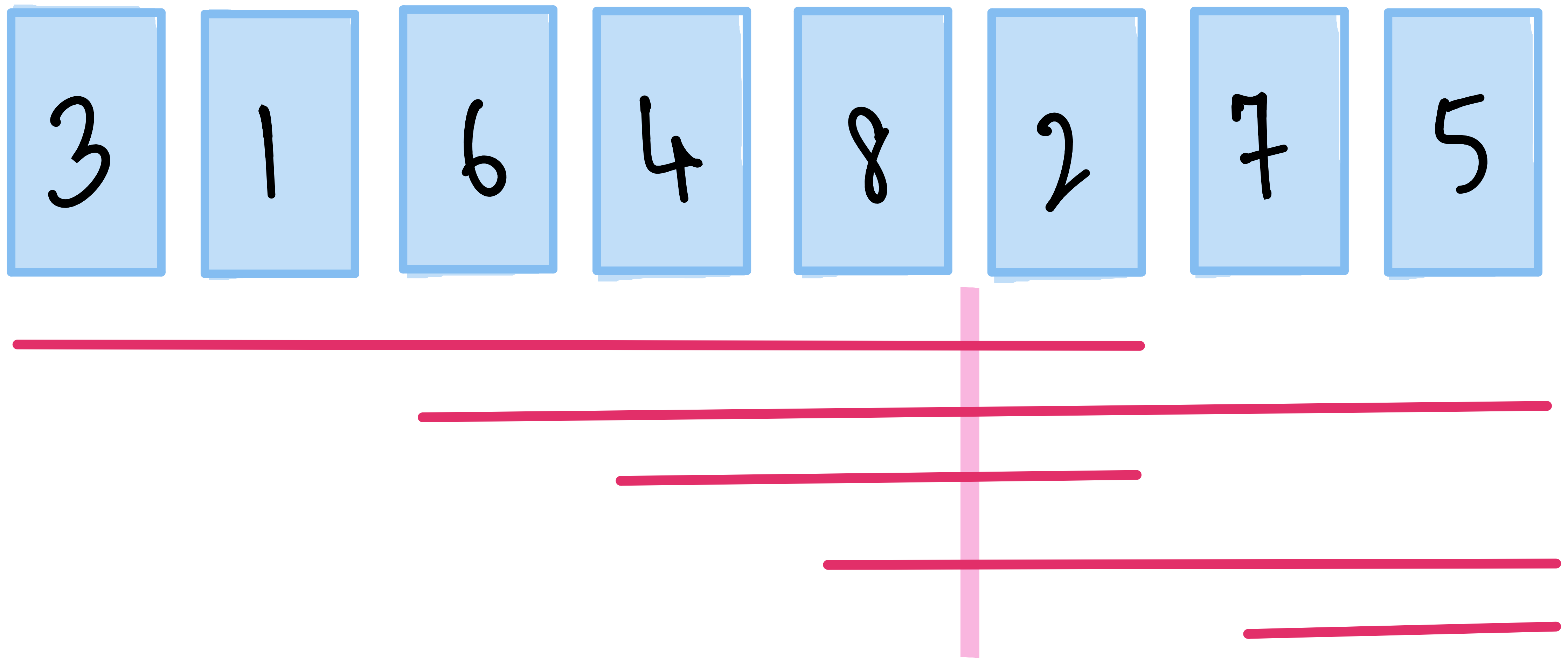

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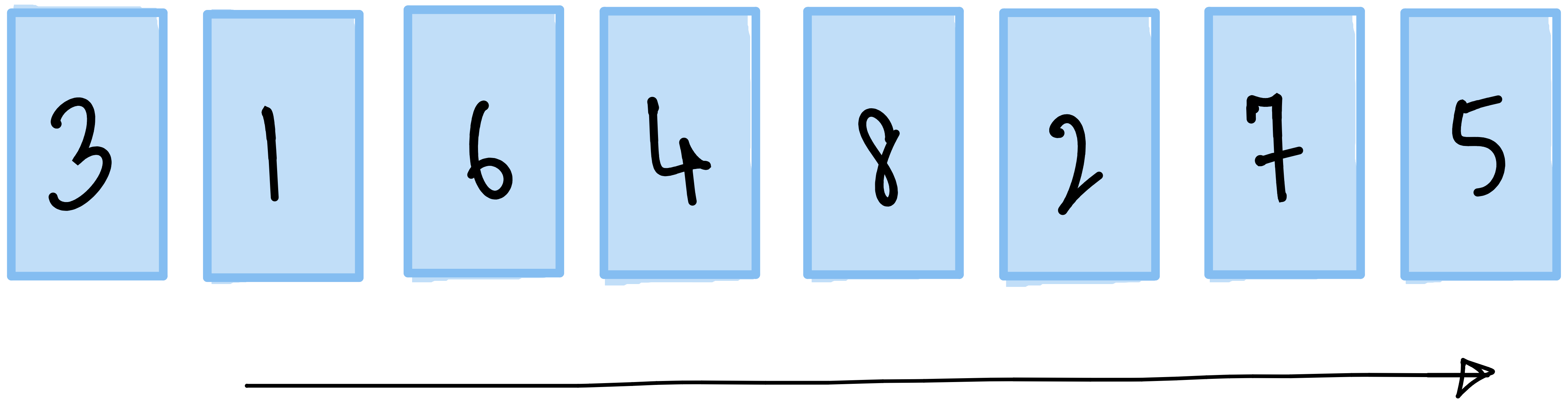
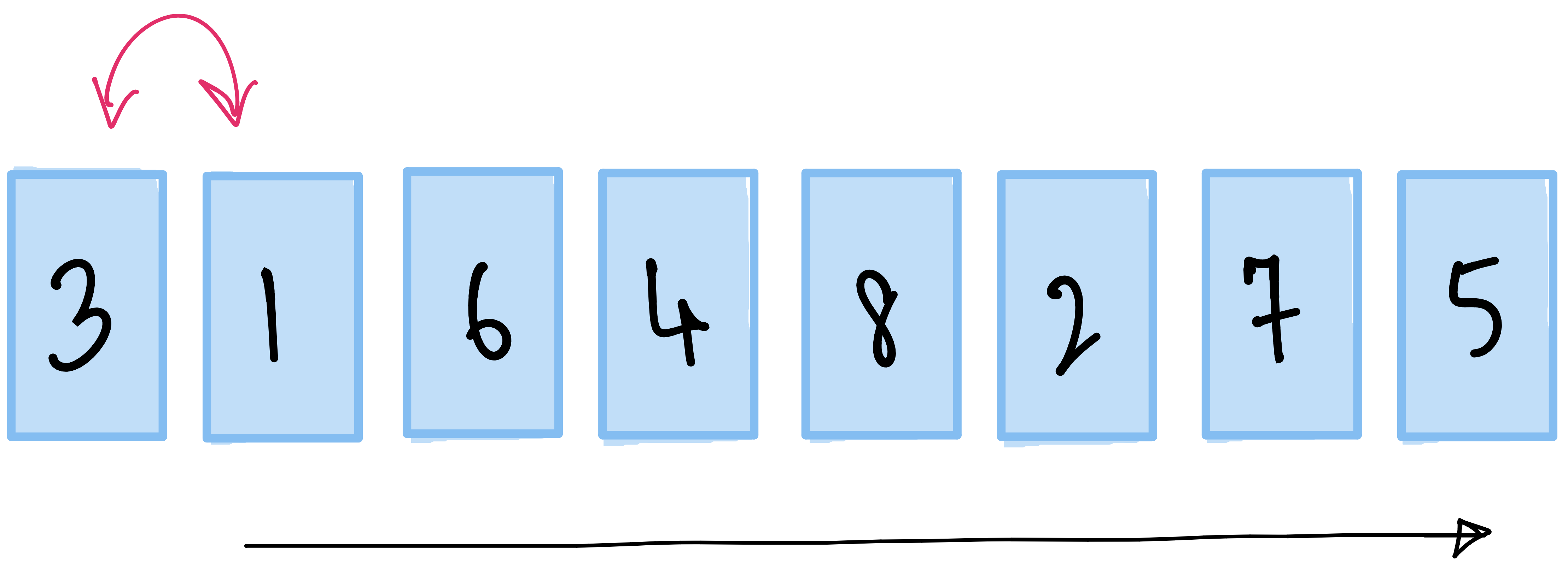


Bubble Sort

One left-to-right pass:

swap adjacent cards if they are

"out of order"



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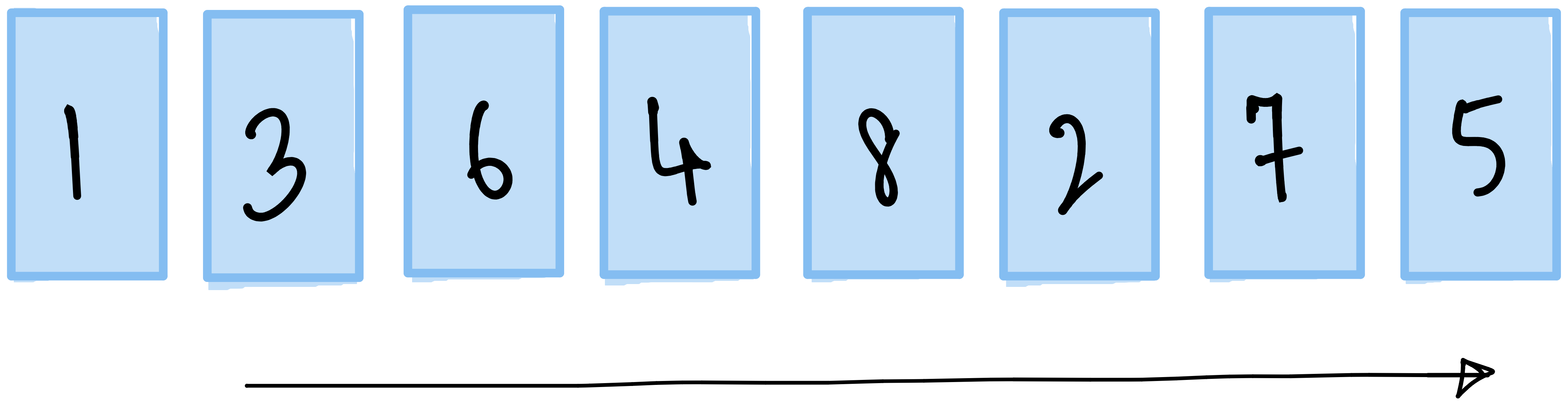

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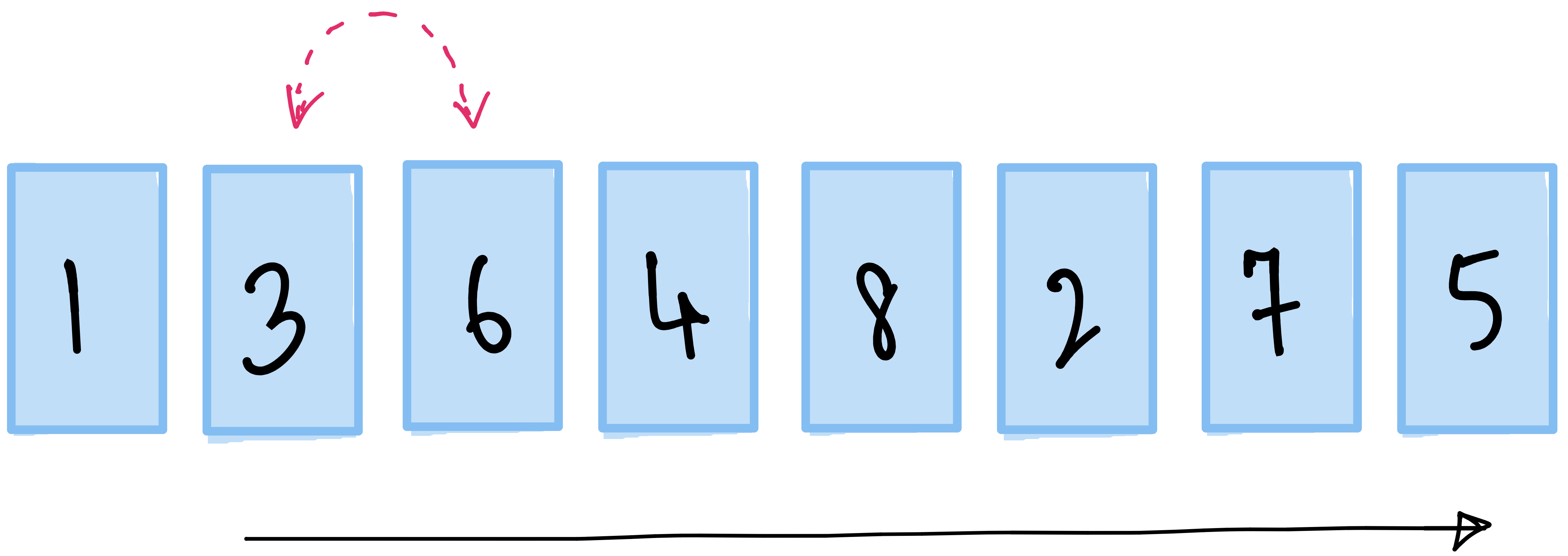

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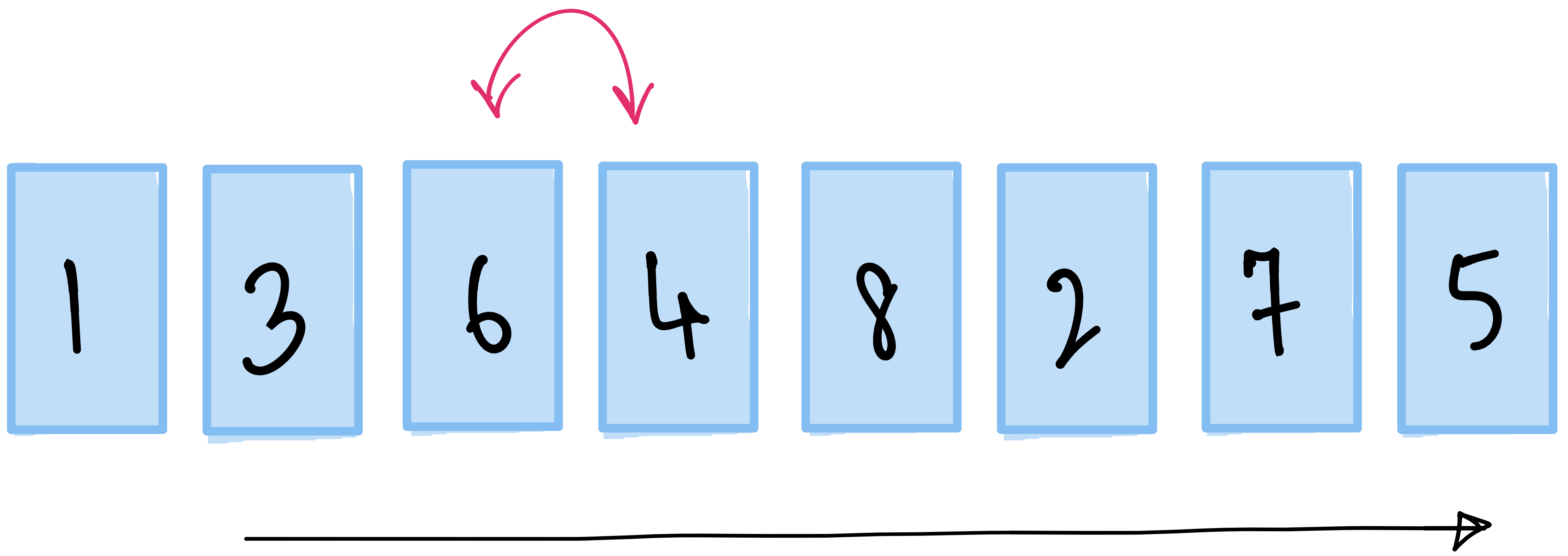

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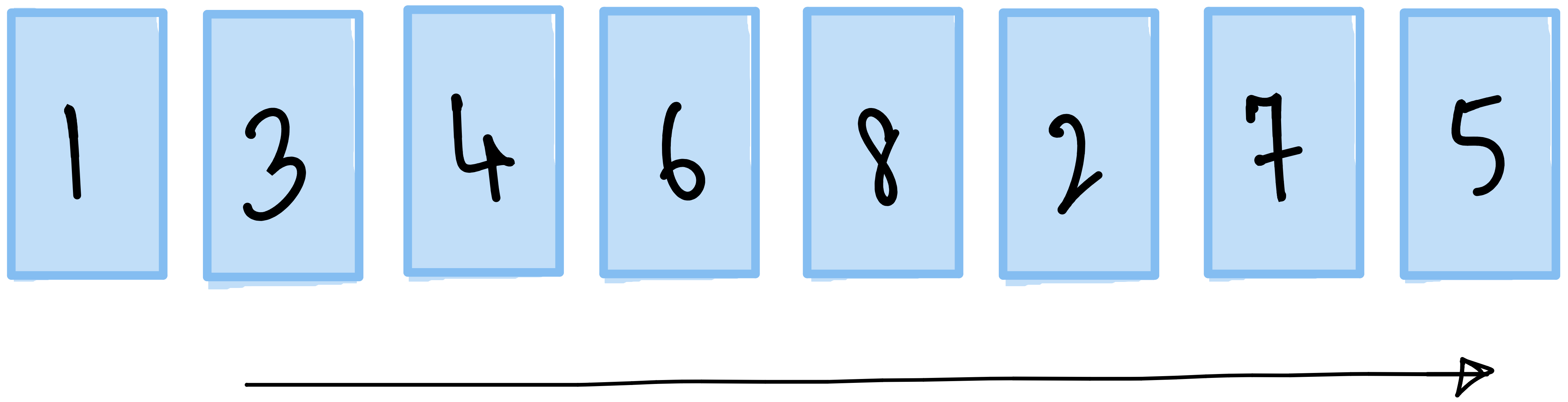

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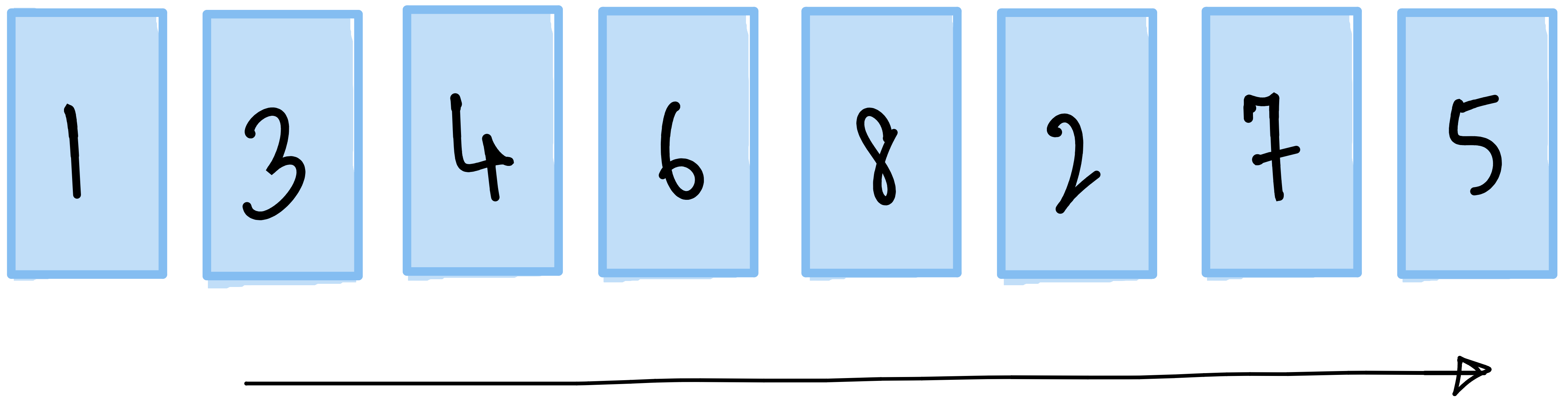

etc

Bubble Sort

One left-to-right pass:

Swap adjacent cards if they are "out of order"

Exercise

Bubble sort passes $= n_{opt}$

Special Cases

$\underbrace{c = 1}_{\text{one color}}$

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one color

LAZY approach: play a card as late as possible.

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Store the LAST PLAYABLE INDEX for each value.

Special Cases $\underbrace{c = 1}_{\text{one color}}$

For any $i \leq N$, we say that the $\underbrace{i^{th} \text{ card}}_{\text{Value: } a_i}$ is useless

if $\exists\, w_1, \dots, w_{n+1} \in \mathbb{N}$ s.t.

- $a_i < w_1 < \dots < w_{n+1} \leq \vartheta$
- $\forall j \in \{i+1, \dots, N\}$, it holds that $a_j \notin \{w_1, \dots, w_{n+1}\}$

Special Cases

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For any $i \le N$, we say that the $\underbrace{i^{th} \text{ card}}_{\text{Value: } a_i}$ is useless

there exist $\geq n+1$ values higher than a_i,

none of which appears after the i^{th} card in σ.

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Why? Claim. Useless cards are never played in a winning sequence.

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one color

Idea

① Filter out the useless cards (how?)

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1. Filter out the useless cards (how?)
2. Among the rest:

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   ignore all cards except the one

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   can play?

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Idea

1. Filter out the useless cards (how?)

2. Among the rest:

   ignore all cards except the one

   which is the last card of a given value in σ^*

   can play?

   $\xleftarrow{\text{yes}}$ play

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Idea

① Filter out the useless cards (how?)

② Among the rest:

ignore all cards except the one

which is the last card of a given value in σ^*

can play?

$\xrightarrow{\text{yes}}$ play

$\xrightarrow{\text{no}}$ store

Special Cases

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one color

Idea

(1) Filter out the useless cards (how?)

(2) Among the rest:

ignore all cards except the one

which is the last card of a given value in σ^*

can play?

yes → play ☆

no → store

☆ also play all the other playable cards

Special Cases

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one color

Claim. If the filtering phase knocks out ALL cards of a given value, there is NO WIN.

Otherwise, the lazy algorithm actually works.