Quiz 0 | Solutions

	Find the k-th element of the sequence	**Adding a new element at the start**	**Adding a new element at the end**	**Removing an element**
Linked List	Proportional to the length of the list	Immediate	Proportional to the length of the list	Proportional to #elements before target OR *immediate* if target given
Array	Immediate	Proportional to the length of the list	Immediate	Proportional to #elements after target

---

1. The Josephus problem is an election method that works by having a group of people stand in a circle. Starting at a predetermined person, one may count around the circle N times. Once the N^{th} person is reached, one should remove them from the circle and have the members close the circle. The process is repeated until only one person is left. That person wins the election. Given the relative ordering of a group of N people, you want to write a program that computers the Josephus winner.

   You decide to store the sequence of N people as a linked list. Your justification for this is:

   (x) It will be efficient to identify the person to be removed in each round.

   (y) Once the person to be removed, say P, has been identified, the deletion of the number in the sequence corresponding to P can be done efficiently.

   Your friend decides to store the sequence of N people as an array. Their justification for this is:

   (p) It will be efficient to identify the person to be removed in each round.

   (q) Once the person to be removed, say P, has been identified, the deletion of the number in the sequence corresponding to P can be done efficiently.

   Which combination of justifications is correct?

   (A) ⭕ (x) and (p) (B) ⭕ (x) and (q) (C) ⭕ (y) and (p) (D) ⭕ (y) and (q)

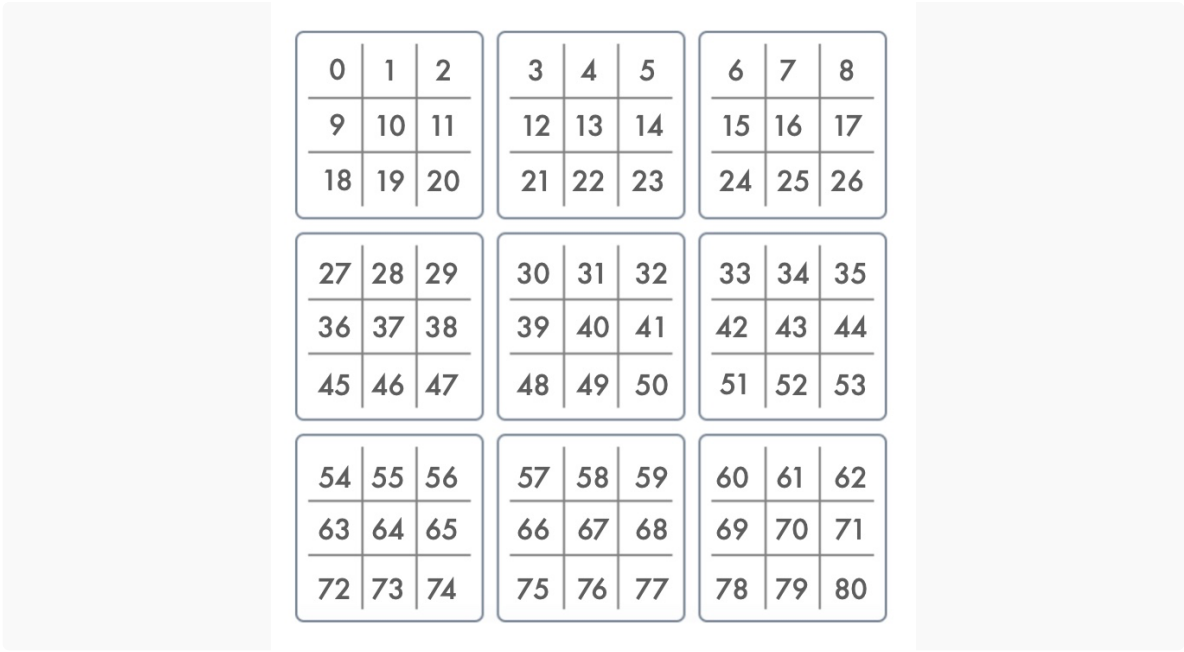
   How confident are you about your answer?

   (A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess

---

2. The game of meta tic-tac-toe is a board game composed of nine Tic Tac Toe boards arranged in a 3-by-3 grid. Players take turns playing in the smaller Tic Tac Toe boards until one of them wins in the larger Tic Tac Toe board. Each small 3-by-3 Tic Tac Toe board is referred to as a local board, and the larger 3-by-3 board is referred to as the global board. At any point in the game, the slots in the board are either empty, or have a **X** or a **O** in them. After $2k$ rounds have been played, there will be k **X**'s and k **O**'s on the board. The exact rules of the game are not relevant to this question, although you might want to look them up later!

   To implement meta tic-tac-toe, we interpret every game state as a sequence of 81 **X**'s, **O**'s and **?**'s, where the **?**'s represent unfilled boxes, and where k^{th} element of the sequence stores the state of the square as identified in the figure below. Notice that the sequence index starts from 0 and the initial state of the game is a sequence of 81 **?**'s.



   The user of your implementation will provide four numbers $1 \leq P, Q, R, S \leq 3$ to indicate that the location of the first move is in the local board on the P-th row and Q-th column, and in the R-th row and S-th column within that board. For example, if $P = 2, Q = 3, R = 2, S = 1$, then the index of the square to be updated is 42.

   Question 2A. In general, given P, Q, R, S as input, what index should you update? Enter your answer on the line below.

$$27(P-1) + 9(R-1) + 3(Q-1) + (S-1)$$

   How confident are you about your answer?

   (A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess

Question 2B. Given that the entire implementation involves only recording updates to cells being played as the game progresses, what data structure will you use to store the sequence that captures the game's state? Assume calculations involving numbers are instantaneous.

(A) ⭕ Linked List (B) ⭕ Array (C) ⭕ Makes no difference

How confident are you about your answer?

(A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess

---

3. This semester, 9 teams will be playing in the IITGN Cricket League, where, every team plays every other team once. There is exactly one match everyday starting from 15th August. You want to keep track of the scores of all the teams as the league progresses. Assume that every match has an outcome and that there are no draws.

   A two-dimensional array stores N^2 numbers in a contiguous $N \times N$ grid. For any $1 \leq i, j, \leq N$, the data in the cell in the i^{th} row and j^{th} column can be updated immediately.

   You use a 9×9 array **A** to represent the outcomes of the league. To begin with, all entries of **A** are 0. On the d^{th} day of the league, suppose team i and team j are playing. Once the match is played, you update:

   - **A**$[i][j]$ to 1 and **A**$[j][i]$ to -1 if team i beats team j, and
   - **A**$[i][j]$ to -1 and **A**$[j][i]$ to 1 if team j beats team i.

   Question 3A. How much time do you need to make these updates?

   (A) ⭕ Immediate

   (B) ⭕ Time proportional to d

   (C) ⭕ Time proportional to the number of matches played by i and j

   (D) ⭕ Time proportional to the number of teams

   How confident are you about your answer?

   (A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess

   Question 3B. At the end of the d^{th} day for some $1 \leq d \leq 72$, you want to know the score of your favourite team (i.e, number of matches won), team k. What best describes the time taken to compute this?

   (A) ⭕ Immediate

   (B) ⭕ Time proportional to d

   (C) ⭕ Time proportional to the number of matches played by k

   (D) ⭕ Time proportional to the number of teams

   How confident are you about your answer?

   (A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess

Question 3C. At the end of the league, you and your friends calculate the score of all 9 teams. Your friends have the scores as shown below.

	1	2	3	4	5	6	7	8	9
A	7	0	4	3	4	6	2	6	5
B	7	7	7	0	0	6	6	2	1
C	8	5	7	5	3	2	1	1	4

Which of your friends definitely has a mistake in their scorekeeping? Check all that apply.

(A) ⭕ A (B) ⭕ B (C) ⭕ C

How confident are you about your answer?

(A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess

Question 3D. You have been asked to design a sequence of 72 matches such that:

a) At the end, each one of the 9 teams would have played all other 8 teams *exactly* once;

b) There is one day when each of the 9 teams have played **exactly five** matches each.

Is it possible to make such a design?

(A) ⭕ Of course: I can make one

(C) ⭕ Impossible and I know it

How confident are you about your answer?

(A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess

Question 3E. You have been asked to design a sequence of 72 matches such that:

a) At the end, each one of the 9 teams would have played all other 8 teams *exactly* once;

b) There is one day when each of the 9 teams have played a **different** number of matches each.

Is it possible to make such a design?

(A) ⭕ Of course: I can make one

(C) ⭕ Impossible and I know it

How confident are you about your answer?

(A) ⭕ quite certain (B) ⭕ reasonably sure (C) ⭕ (educated) guess